Part I : Age

General definition

Age = time elapsed since a given origin !

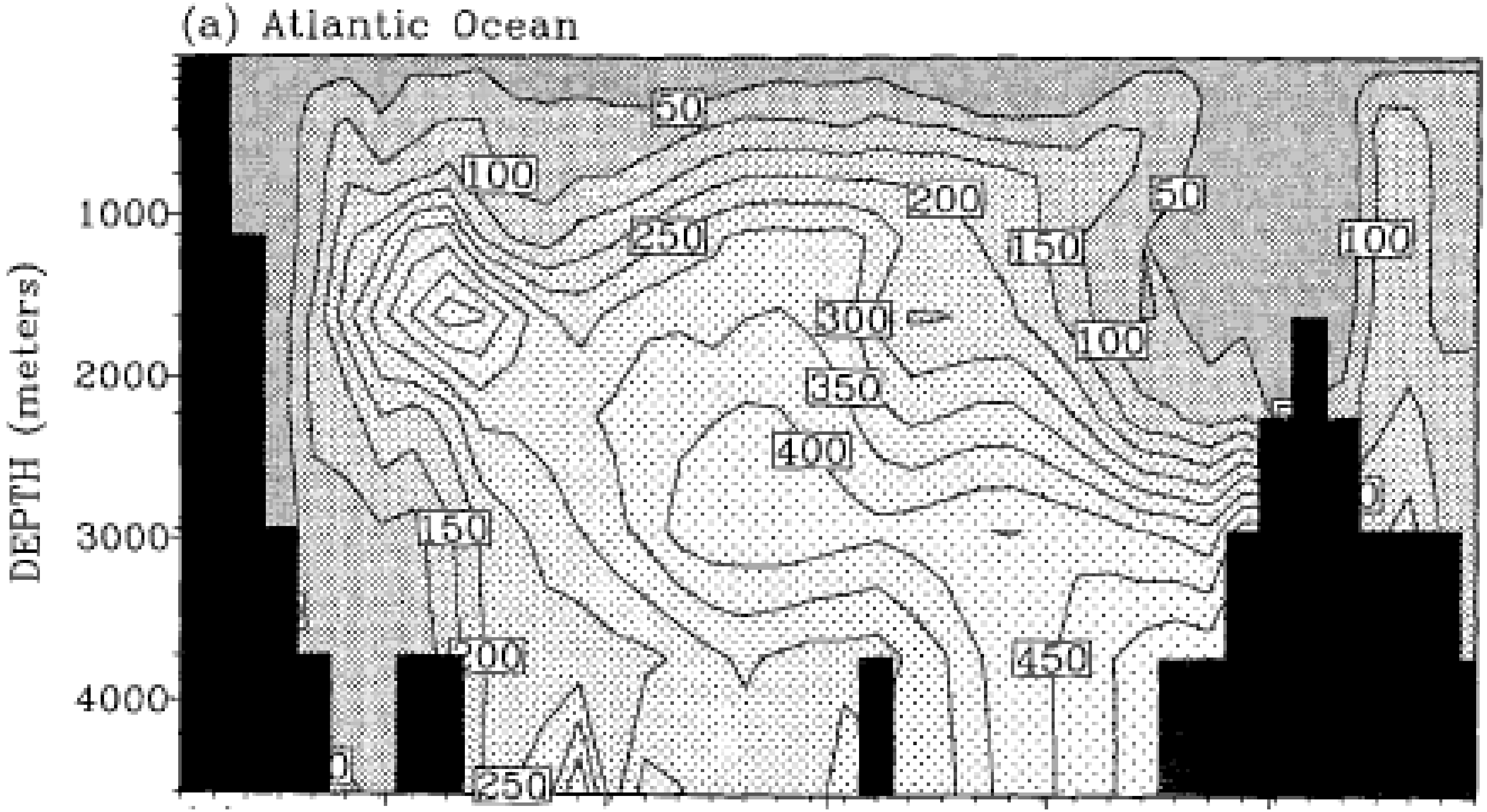
$\Rightarrow$ Interpretation depends on the origin.

Ex. :

- Ventilation rate of the ocean
- Age of tracers (e.g. technetium-99)
- ...

Ventilation rate

Age = time elapsed since leaving
the surface mixed layer



See for instance England, 1995, Journal of Physical Oceanography, 25, 2756-2777

Inferring shelf sea circulation

Age = time elapsed since leaving the source

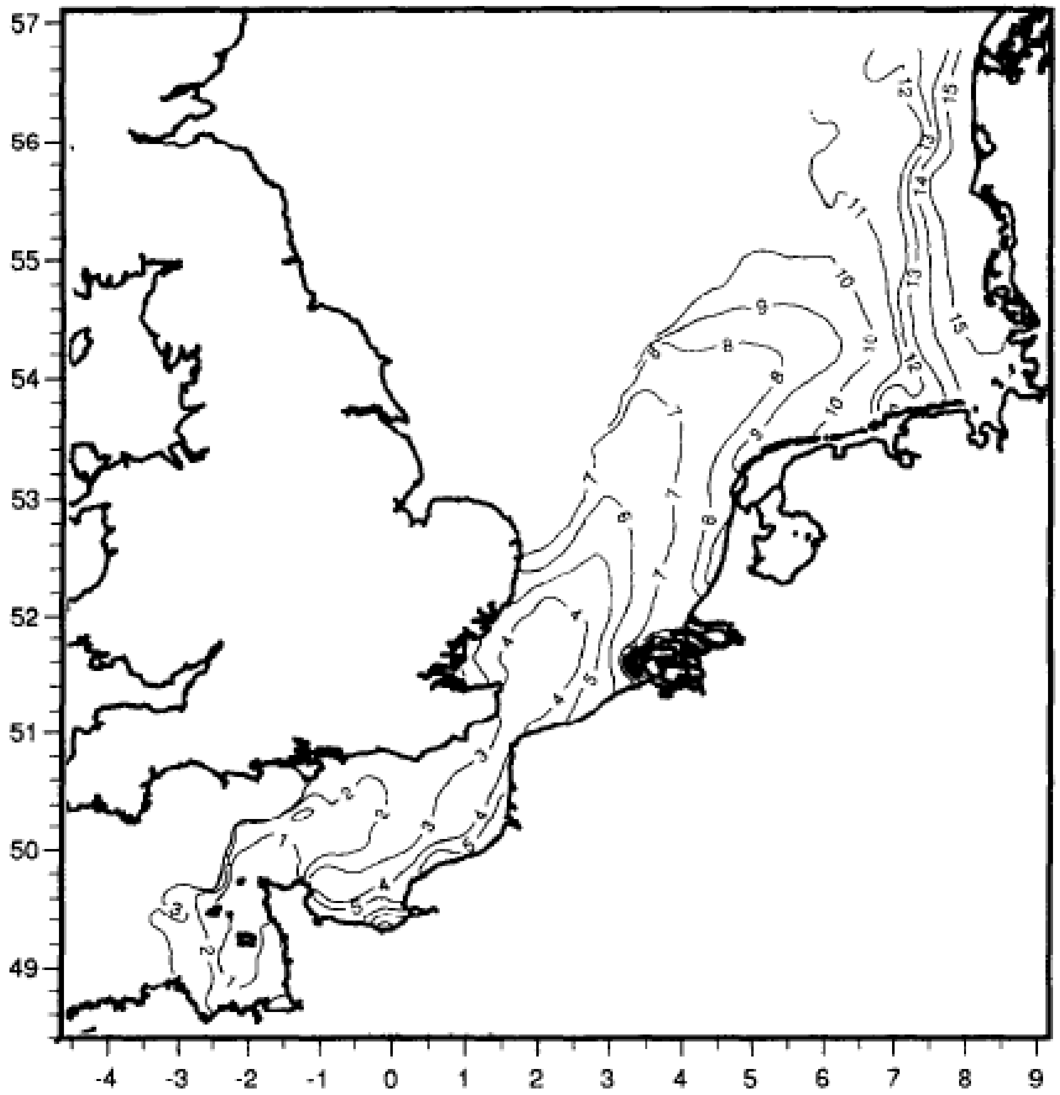


Fig. 2. Transit time from La Hague (in months).

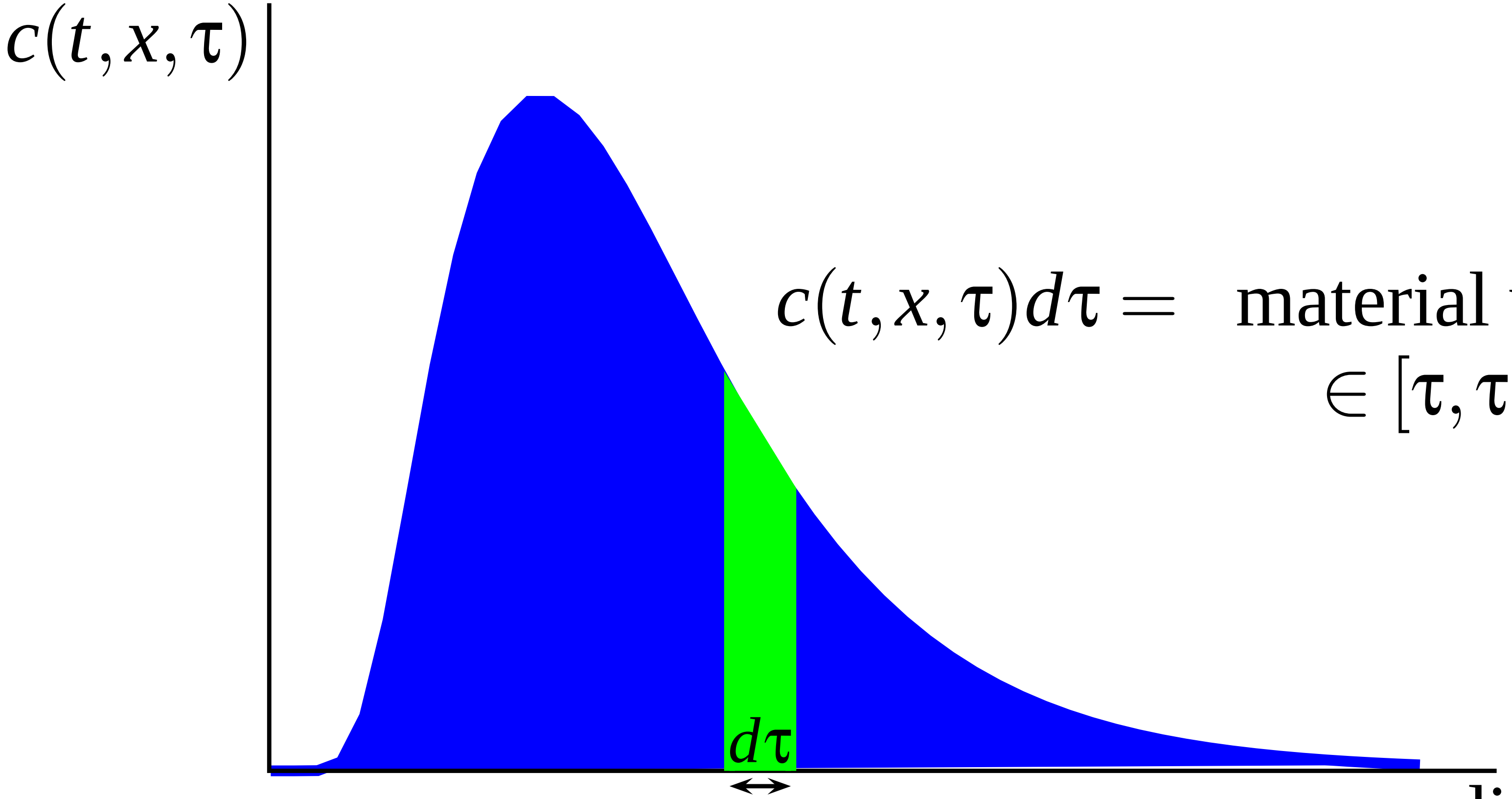
Salomon et al. 1995, Journal of Marine Systems, 6, 515-527

CART

Constituent oriented **A**ge and **R**esidence time **T**heory

- Advection, mixing, and production/destruction properly accounted for;
- time- and position-dependent age;
- the age of every constituent can be evaluated;
- Eulerian formalism;
- mainly for numerical models, (some aspects may apply to field data).

Concentration distribution function



$c(t,x,\tau)d\tau =$ material with age $\in [\tau, \tau + d\tau]$

τ : age dimension

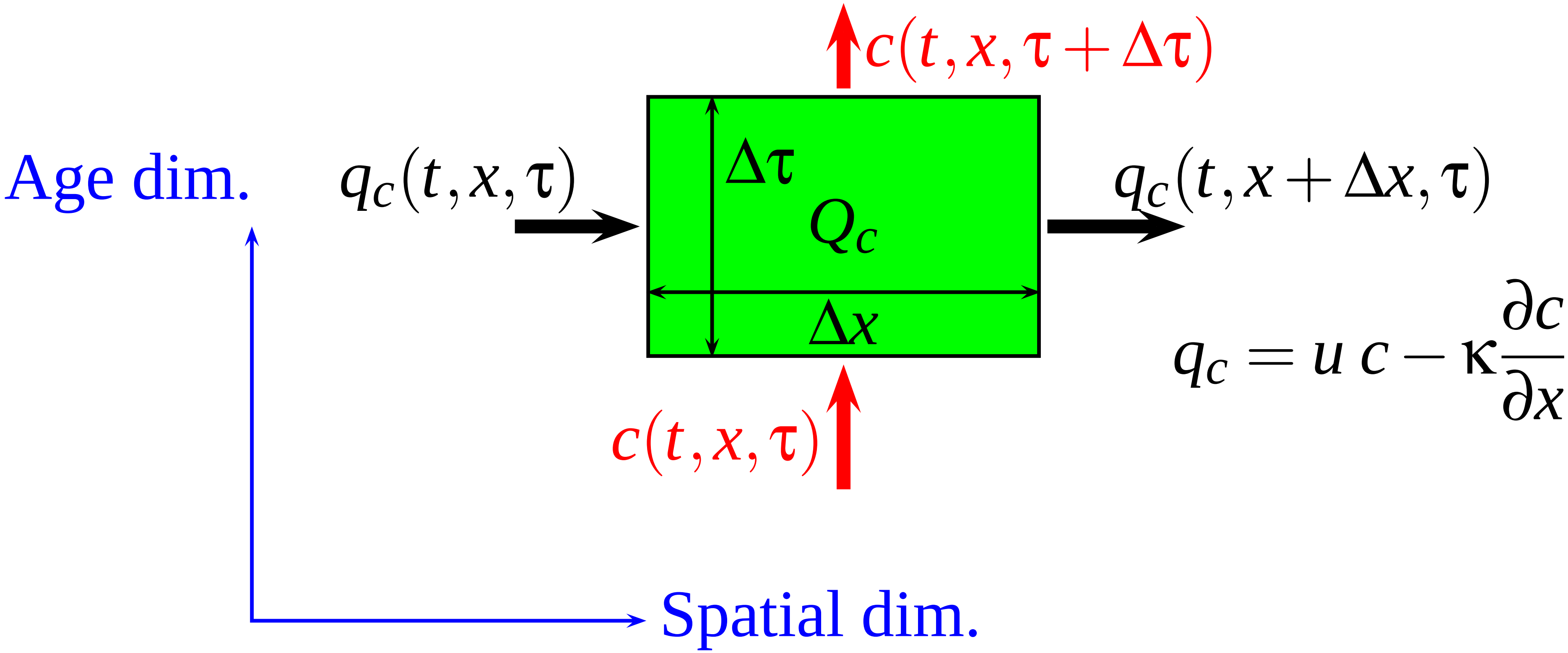
$$C(t,x) = \int_0^\infty c(t,x,\tau)d\tau$$

where $\tau =$ age dimension

Mass budget : equation for c

Budget in *space* $\times$ *age*

$$\frac{\partial c}{\partial t} = Q_c - \nabla \cdot \mathbf{q}_c - \frac{\partial c}{\partial \tau}$$



Consistency check

$$\frac{\partial c}{\partial t} + \mathbf{v} \cdot \nabla c + \frac{\partial c}{\partial \tau} = \nabla \cdot (\mathbf{K} \cdot \nabla c)$$

to be solved in a 5-dimensional space

$$\int_0^\infty \ldots d\tau \qquad \Rightarrow$$

$$\boxed{\frac{\partial C}{\partial t} + \mathbf{v} \cdot \nabla C = \nabla \cdot (\mathbf{K} \cdot \nabla C)}$$

since

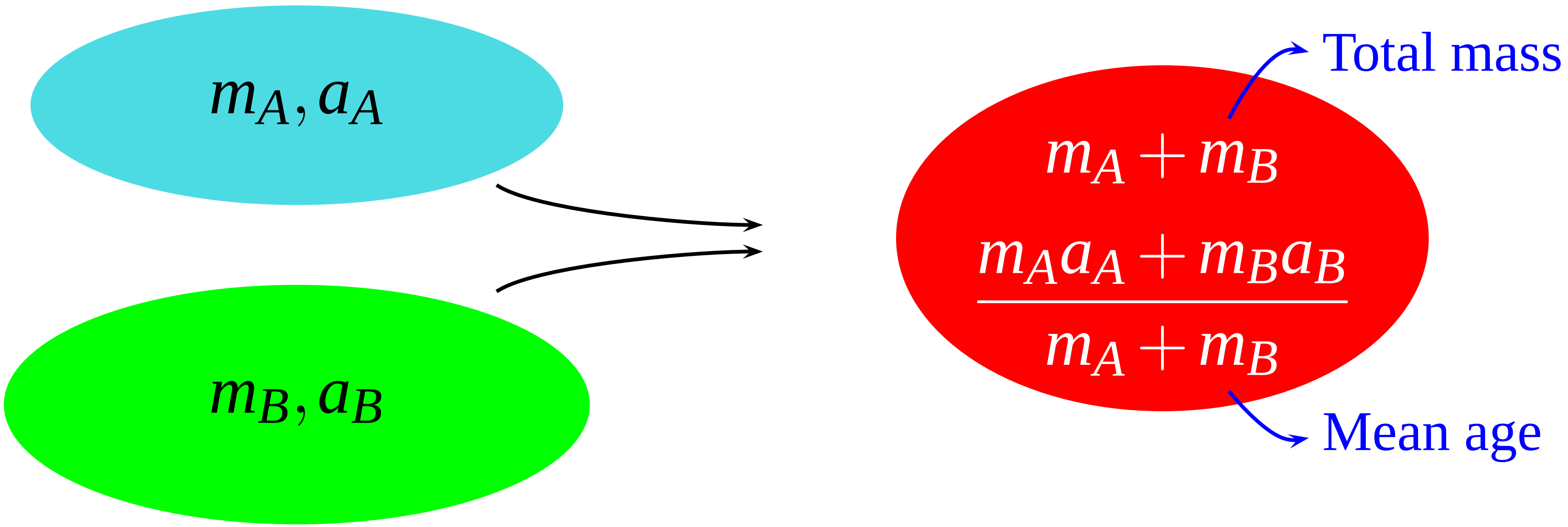
$$\int_0^\infty \frac{\partial c}{\partial \tau}(t, \mathbf{x}, \tau) d\tau = c(t, \mathbf{x}, \tau)\Big]_0^\infty = 0$$

Age averaging hypothesis

Mean age :

$$a(t,\mathbf{x}) = \frac{1}{C(t,\mathbf{x})}\int_0^\infty \tau c(t,\mathbf{x},\tau)\,d\tau$$

Consistent with



The mean age is not additive !

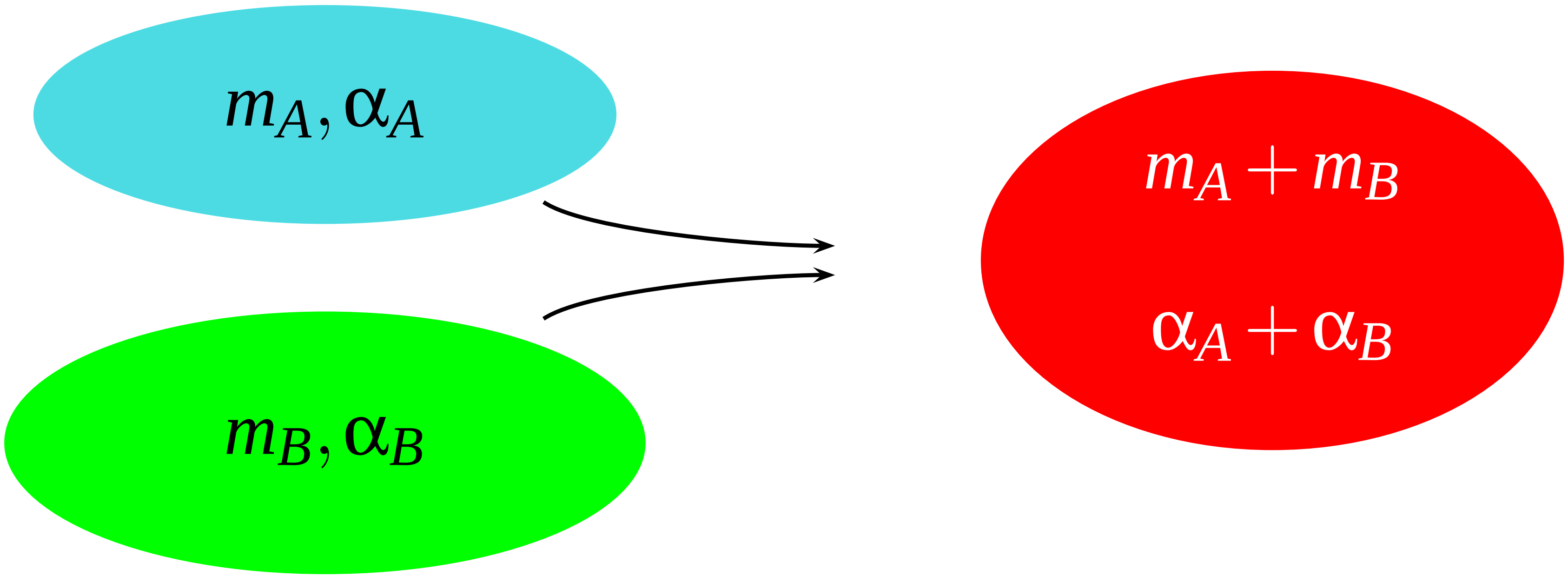
CART - 2007

Age concentration

Mean age :

$$\alpha(t,\mathbf{x}) = a(t,\mathbf{x})C(t,\mathbf{x}) = \int_0^\infty \tau c(t,\mathbf{x},\tau)d\tau$$

Consistent with



The age concentration is additive !

Evolution equation for α

$$\frac{\partial c}{\partial t} + \mathbf{v}\cdot\nabla c + \frac{\partial c}{\partial \tau} = \nabla\cdot(\mathbf{K}\cdot\nabla c)$$

$$\int_0^\infty \tau \ldots d\tau \qquad \Rightarrow$$

$$\boxed{\frac{\partial \alpha}{\partial t} + \mathbf{v}\cdot\nabla\alpha = C + \nabla\cdot(\mathbf{K}\cdot\nabla\alpha)}$$

since

$$\int_0^\infty \tau \frac{\partial c}{\partial \tau}(t,\mathbf{x},\tau)\,d\tau = \Big[\tau c(t,\mathbf{x},\tau)\Big]_0^\infty - \int_0^\infty c(t,\mathbf{x},\tau)\,d\tau = -C(t,\mathbf{x})$$

Computation strategy

Solve

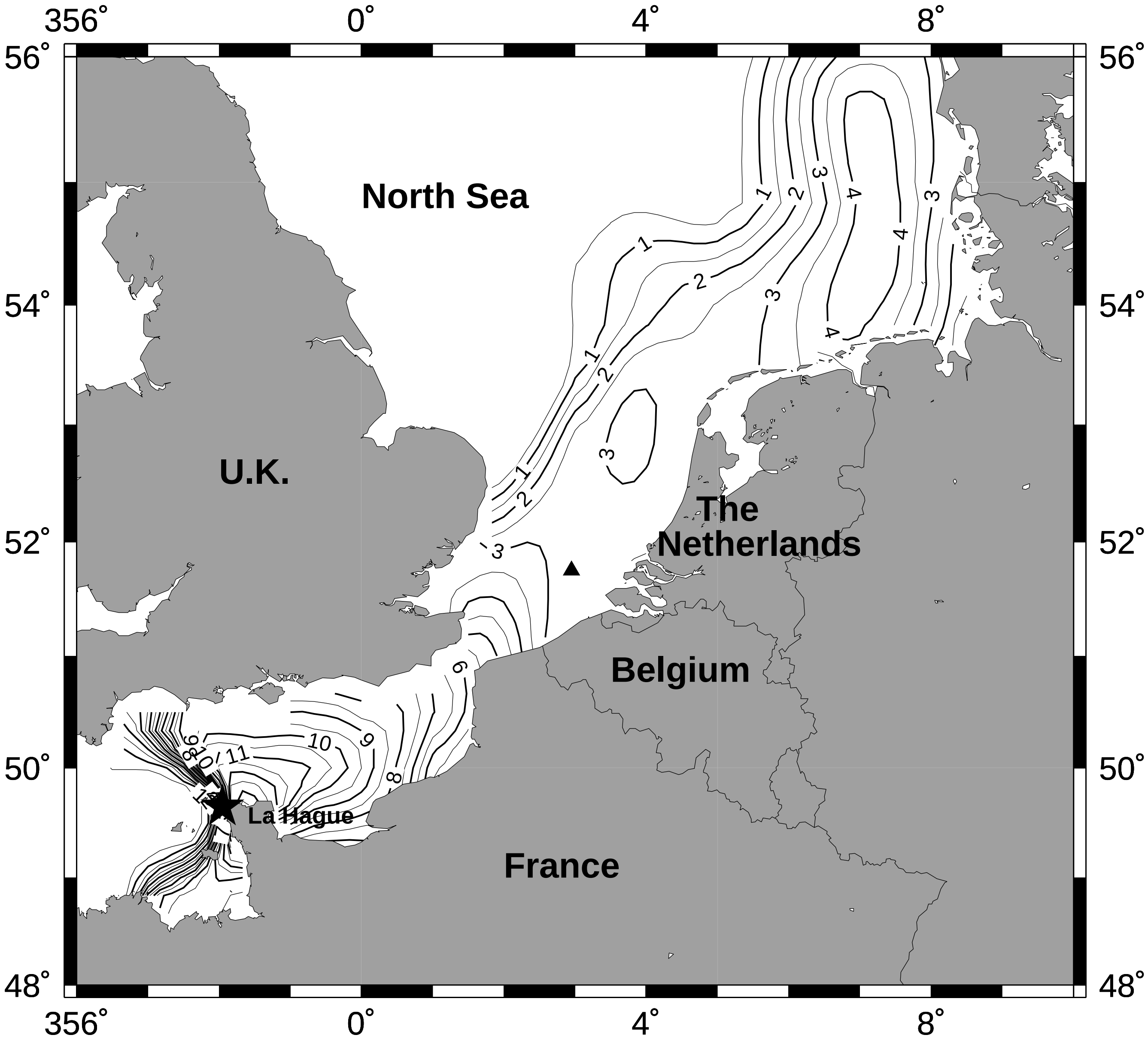
$$\frac{\partial c}{\partial t} + \mathbf{v} \cdot \nabla c + \frac{\partial c}{\partial \tau} = \nabla \cdot (\mathbf{K} \cdot \nabla c)$$

or
$$\begin{cases} \dfrac{\partial C}{\partial t} + \mathbf{v} \cdot \nabla C + = \nabla \cdot (\mathbf{K} \cdot \nabla C) \\ \dfrac{\partial \alpha}{\partial t} + \mathbf{v} \cdot \nabla \alpha = \boxed{C} + \nabla \cdot (\mathbf{K} \cdot \nabla \alpha) \end{cases} \Rightarrow \quad a = \frac{\alpha}{C}$$

+ B.C. defining the 'birth' of the particles and/or tagging the water mass.

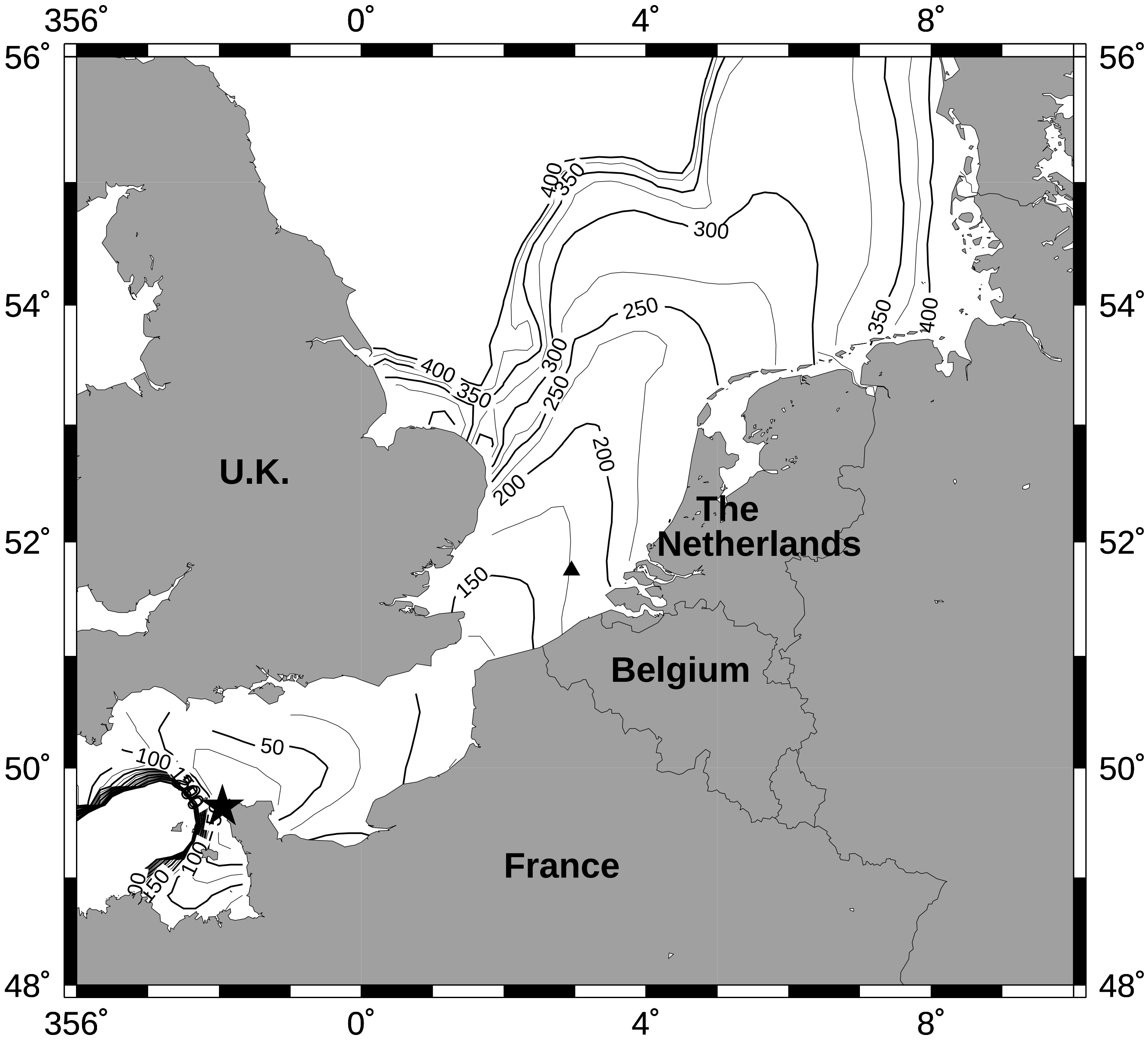
All advection-diffusion operators of the same form !
⇒ readily implemented in numerical models

^{99}Tc in the North Sea - Conc.



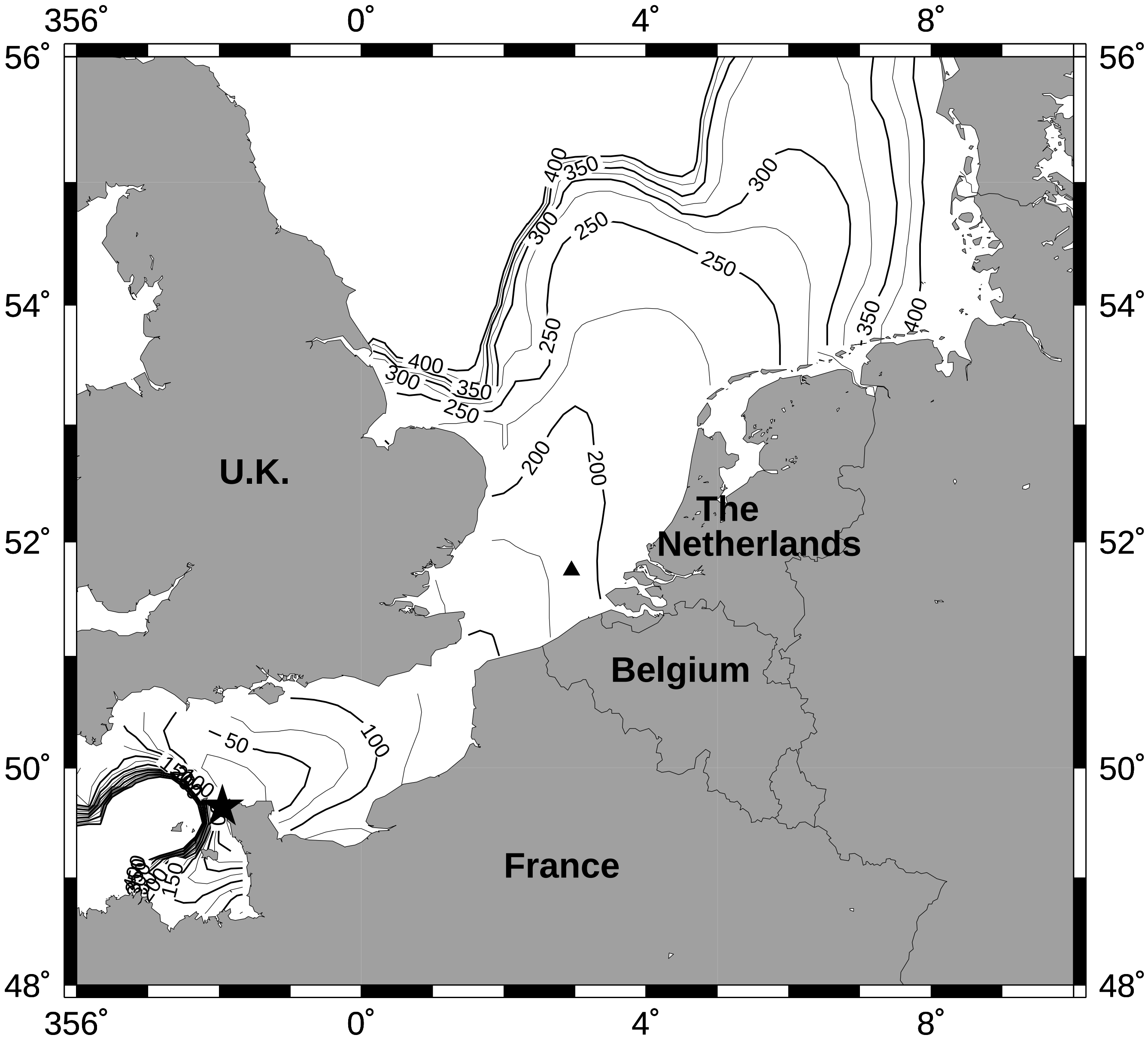
March
1985
Bq/m^3

^{99}Tc in the North Sea - Age



March
1985
days

Artificial tracer - Age



1D :

$$a \sim \frac{x}{u}$$

March 1985

days

Age of the water

Concentration of the water ≈ 1

$$\Rightarrow \alpha = aC \approx a$$

and

$$\frac{\partial a}{\partial t} + \mathbf{v} \cdot \nabla a = 1 + \nabla \cdot (\mathbf{K} \cdot \nabla a)$$

(England, 1995, Journal of Physical Oceanography, 25, 2756-2777)

Dynamic tracers

$$\frac{\partial c}{\partial t} + \mathbf{v} \cdot \nabla c + \frac{\partial c}{\partial \tau} = \nabla \cdot (\mathbf{K} \cdot \nabla c) + p(\tau) - d(\tau)$$

$$\frac{\partial C}{\partial t} + \mathbf{v} \cdot \nabla C = \nabla \cdot (\mathbf{K} \cdot \nabla C) + \int_0^\infty [p(\tau) - d(\tau)] d\tau$$

$$\frac{\partial \alpha}{\partial t} + \mathbf{v} \cdot \nabla \alpha = C + \nabla \cdot (\mathbf{K} \cdot \nabla \alpha) + \int_0^\infty [p(\tau) - d(\tau)] \tau d\tau$$

$$\Rightarrow \qquad a(t, \mathbf{x}, \tau) = \frac{\alpha(t, \mathbf{x}, \tau)}{C(t, \mathbf{x}, \tau)}$$

Age of a radioactive tracer

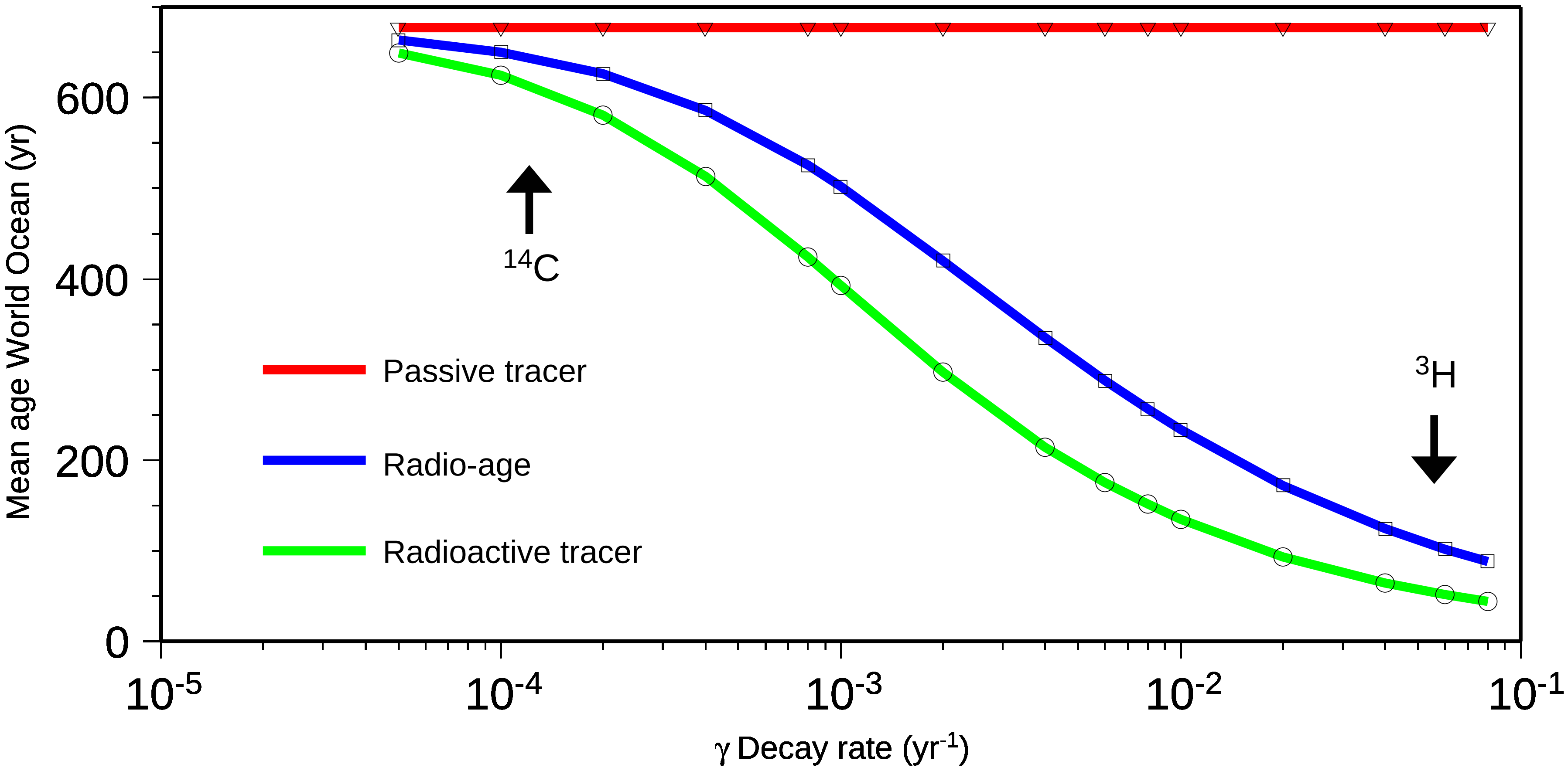
$$p(\tau) = 0, \qquad d(\tau) = -\gamma c_\gamma(\tau)$$

$$\Rightarrow \quad -\gamma \int_0^\infty c_\gamma(\tau) d\tau = -\gamma C_\gamma, \quad -\gamma \int_0^\infty c_\gamma(\tau) \tau d\tau = -\gamma \alpha_\gamma$$

$$\begin{cases} \dfrac{\partial C_\gamma}{\partial t} + \mathbf{v} \cdot \nabla C_\gamma = -\gamma C_\gamma + \nabla \cdot (\mathbf{K} \cdot \nabla C_\gamma) \\ \dfrac{\partial \alpha_\gamma}{\partial t} + \mathbf{v} \cdot \nabla \alpha_\gamma = C_\gamma - \gamma \alpha + \nabla \cdot (\mathbf{K} \cdot \nabla \alpha_\gamma) \end{cases}$$

Radio-age in the world ocean

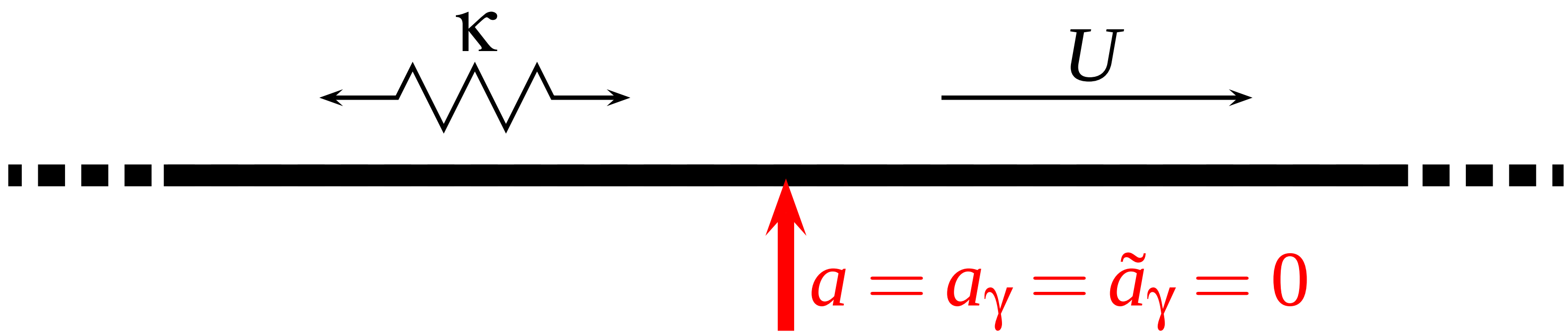
$$\begin{cases} C_0 \\ C_\gamma \end{cases} : \quad a_\gamma \quad < \quad \tilde{a}_\gamma = \frac{1}{\gamma} \ln \frac{C_0}{C_\gamma} \quad < \quad a_0$$



Without mixing, all ages would be equal !

CART - 2007

1D idealized problem

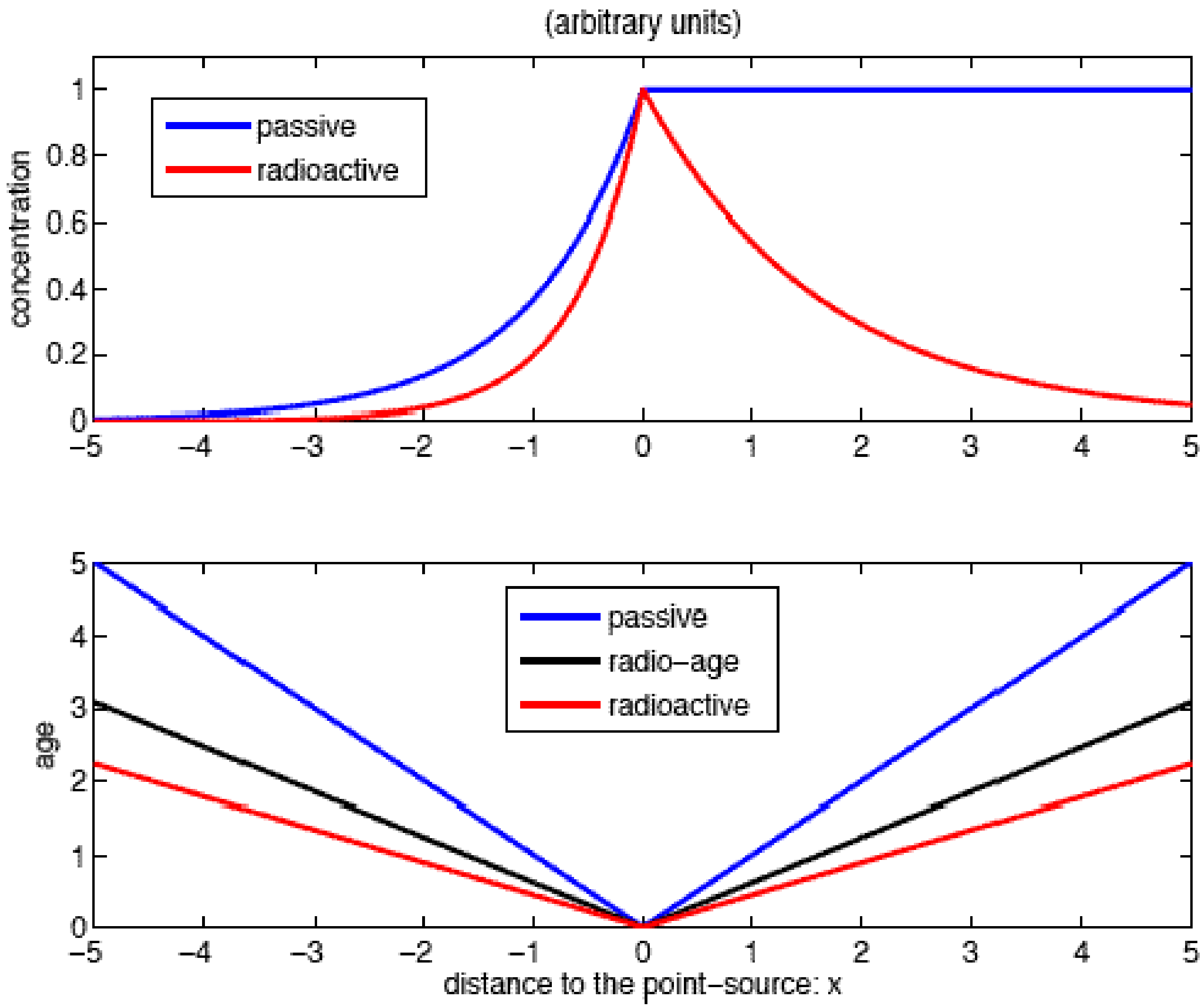


$$U\frac{\partial C_\gamma}{\partial x} = -\gamma C_\gamma + \kappa\frac{\partial^2 C_\gamma}{\partial x^2}$$

$$U\frac{\partial \alpha_\gamma}{\partial x} = C_\gamma - \gamma\alpha_\gamma + \kappa\frac{\partial^2 \alpha_\gamma}{\partial x^2}$$

$$\Rightarrow \qquad a = \frac{\alpha_0}{C_0}, \qquad a_\gamma = \frac{\alpha_\gamma}{C_\gamma}, \qquad \tilde{a}_\gamma = \frac{1}{\gamma}\ln\frac{C_0}{C_\gamma}$$

Steady-state solution



$$a = \frac{|x|}{U}$$

$$\frac{\tilde{a}_\gamma}{a} = \frac{2}{\sqrt{1+4Je^{-1}}+1}$$

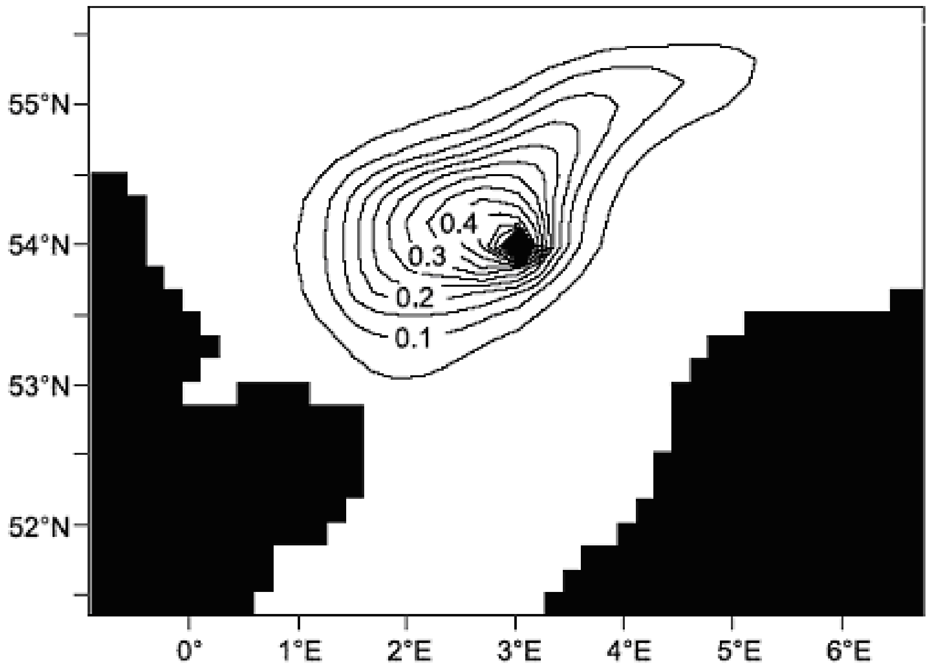
$$\frac{a_\gamma}{a} = \frac{1}{\sqrt{1+4Je^{-1}}}$$

$$Je = \frac{U^2}{\kappa\gamma}$$

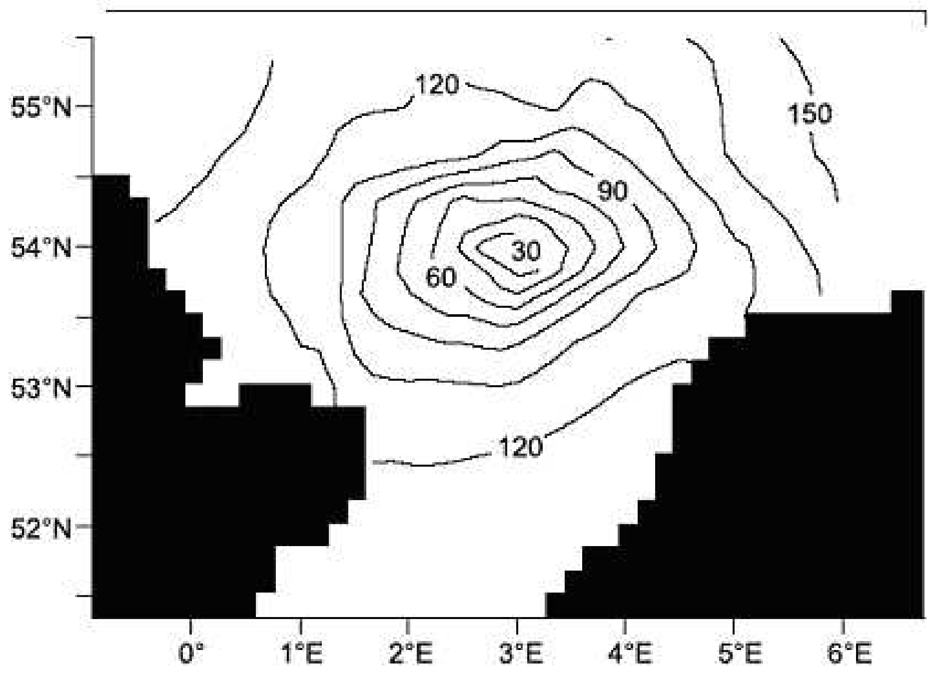
Symmetry !

Symmetry of the age field

Concentration



Radio-age



The tracer concentration reflects the direction of the advection, but the age does not!

See also the Lagrangian approach by Hall and Haine

(2004, Journal of Marine Systems, 48, 51-59)

Sediment dynamics

Aim : diagnose sediment resuspension and transport in the Belgian Coastal Zone.

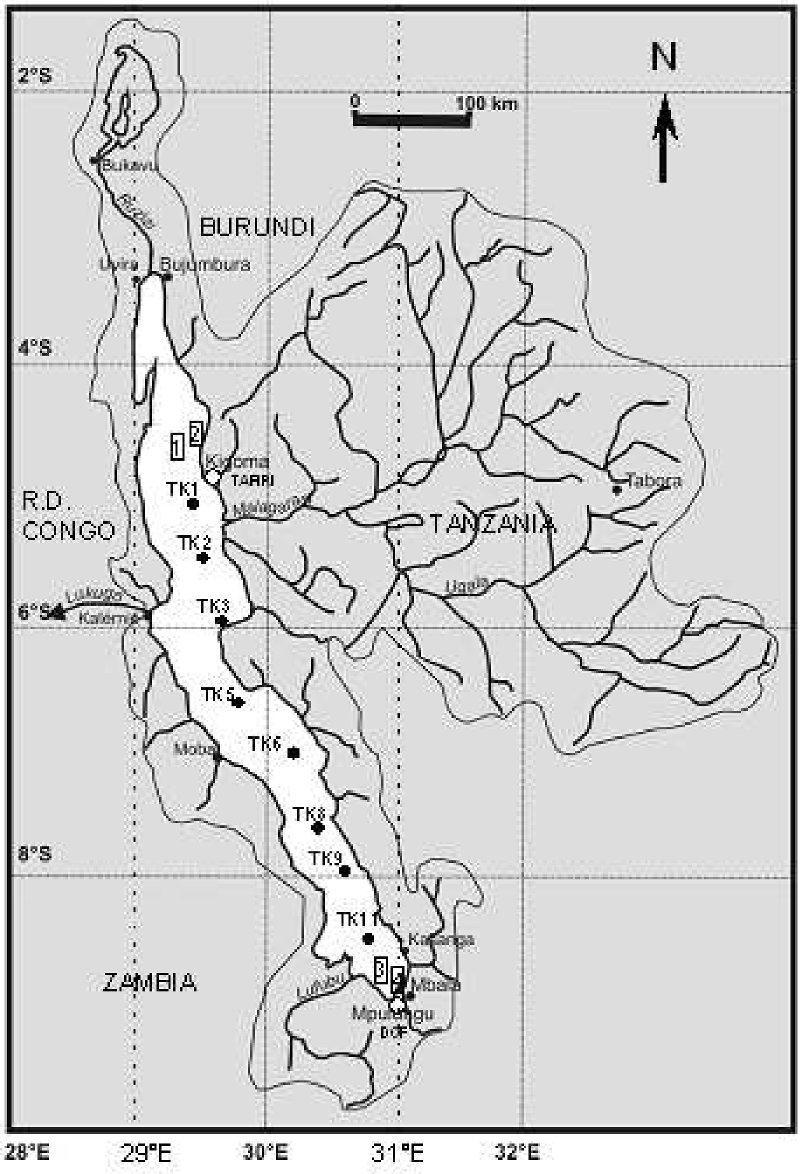
Model : 3D hydrodynamic model nested in a shelf (Extended North Sea) model with sediment module (water column + active bottom layer + parent bottom layer

Age :

1. 'Resuspension age' : reset at resuspension time
   $\Rightarrow$ Time elapsed since resuspension.
2. 'Transport age' : reset when crossing a ref. line
   $\Rightarrow$ Rate of horizontal transport.

Animation : resupension age

Tanganyika’s epilimnion water



- Finite-element, reduced-gravity model
- New water and nutrients brought from the hypolimnion
- Dry season (April-August) with strong winds from south-east; wet season (September-March) with weak winds.

CART - 2007

Animation : depth of thermocline

conc. of hypolimnion water in surface layer

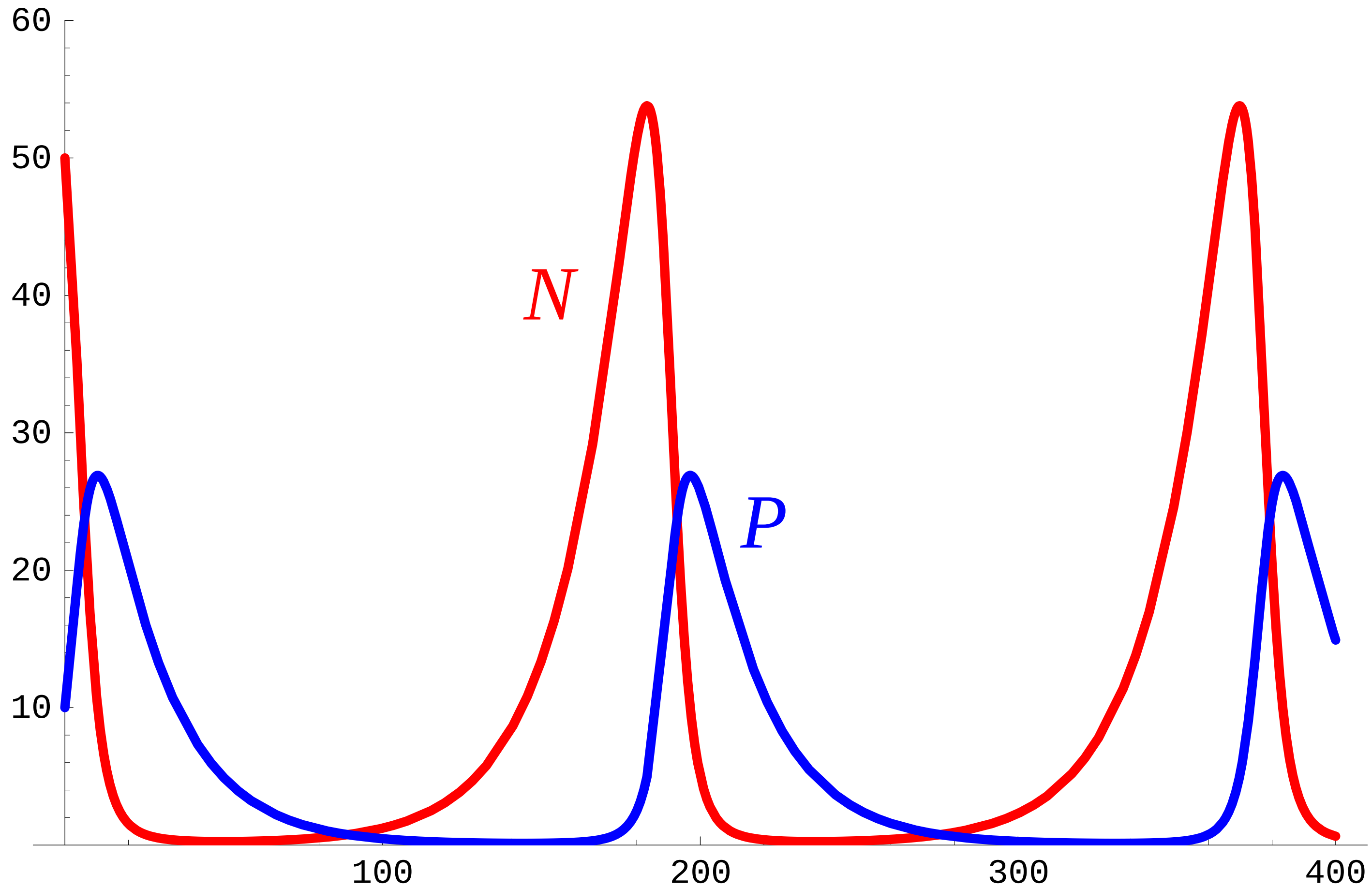
age of hypolimnion water

Lotka-Volterra system

- $N(t)$: prey population
- $P(t)$: predator population

$$\begin{cases} \dfrac{dN}{dt} = \mu N - a\,N\,P \\ \dfrac{dP}{dt} = b\,N\,P - m\,P \end{cases}$$

Lotka-Volterra system (2)



⇒ Cycle !

Lotka-Volterra - age

$$\frac{dN}{dt} = \mu N - a P N$$

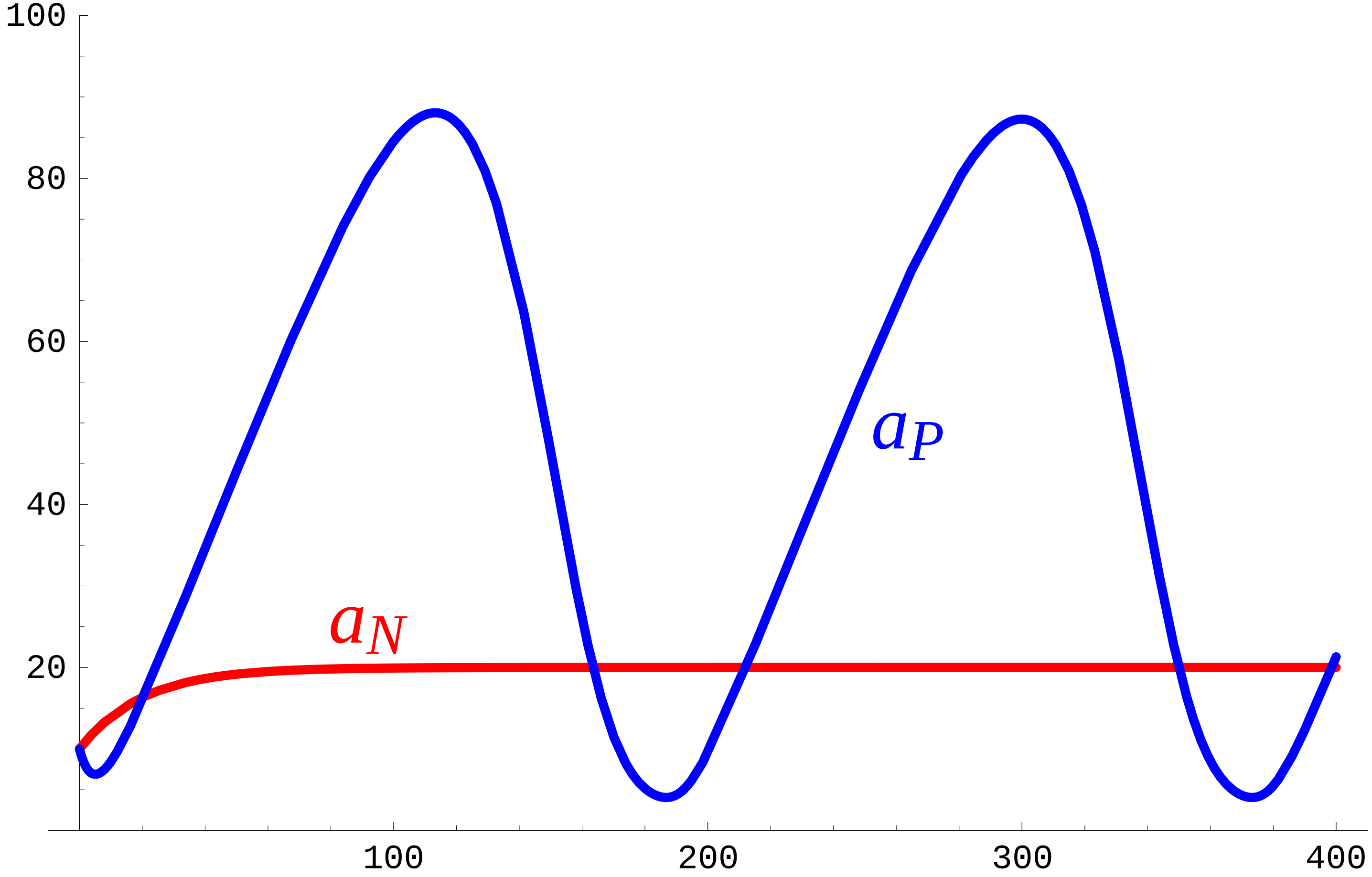
$$\alpha_N = N a_N \quad : \quad \frac{d\alpha_N}{dt} = N - a P \alpha_N$$

$$\frac{dP}{dt} = b P N - m P$$

$$\alpha_P = P a_P \quad : \quad \frac{d\alpha_P}{dt} = P - m \alpha_P$$

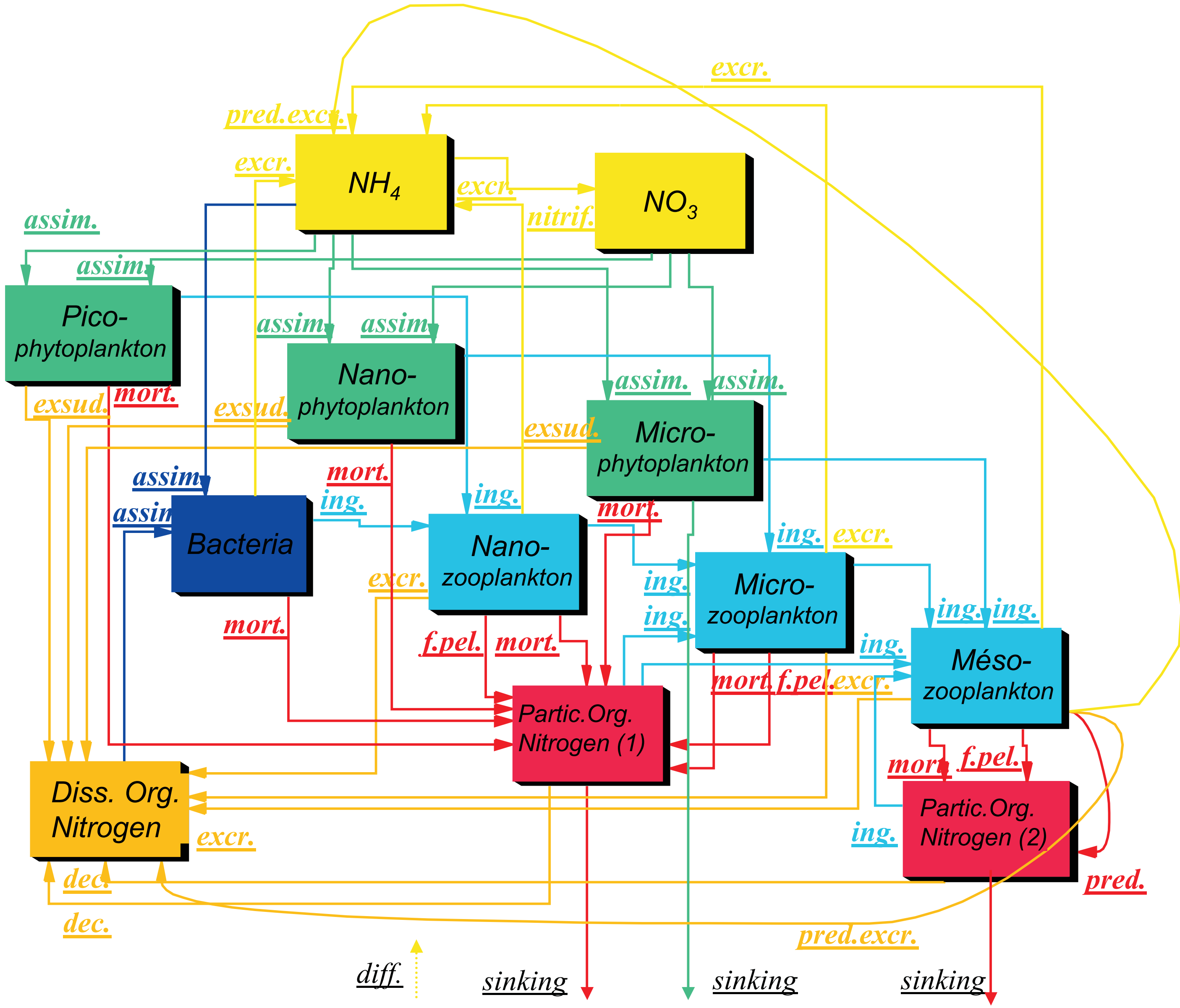
4 coupled ODE !

Lotka-Volterra - age (2)



$$\lim_{t \to +\infty} a_N(t) = \frac{1}{\mu}$$

Ecosystem model - Ligurian sea



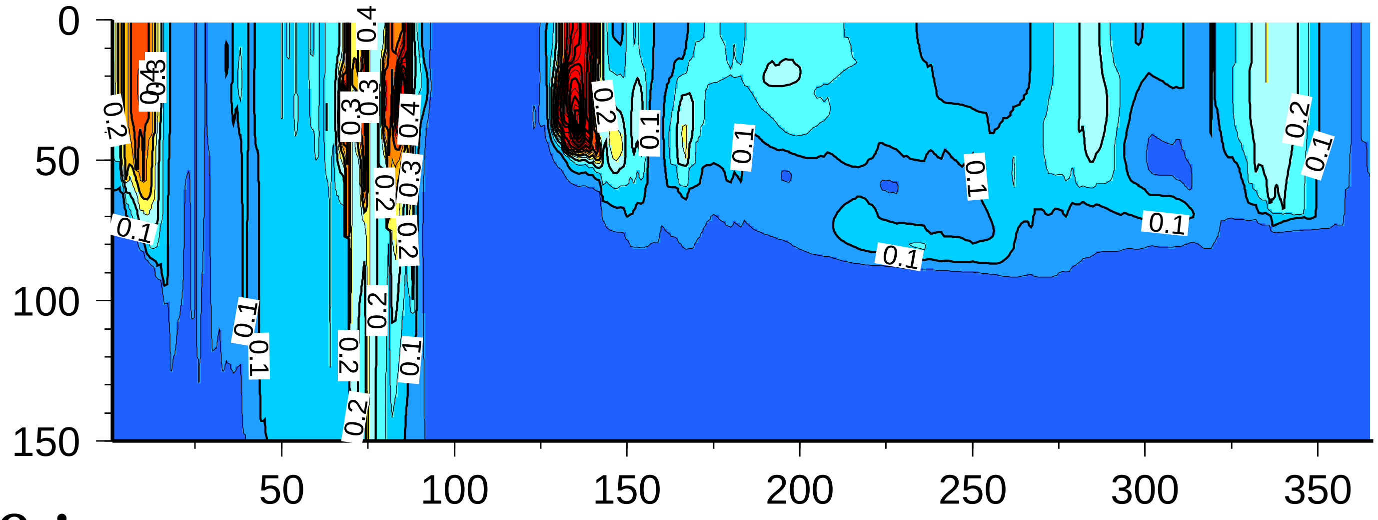
Rate of turnover of nitrogen

- Age of NO_3 and NH_4 = 0 at the time of assimilation
- Compute the age of all other compartments

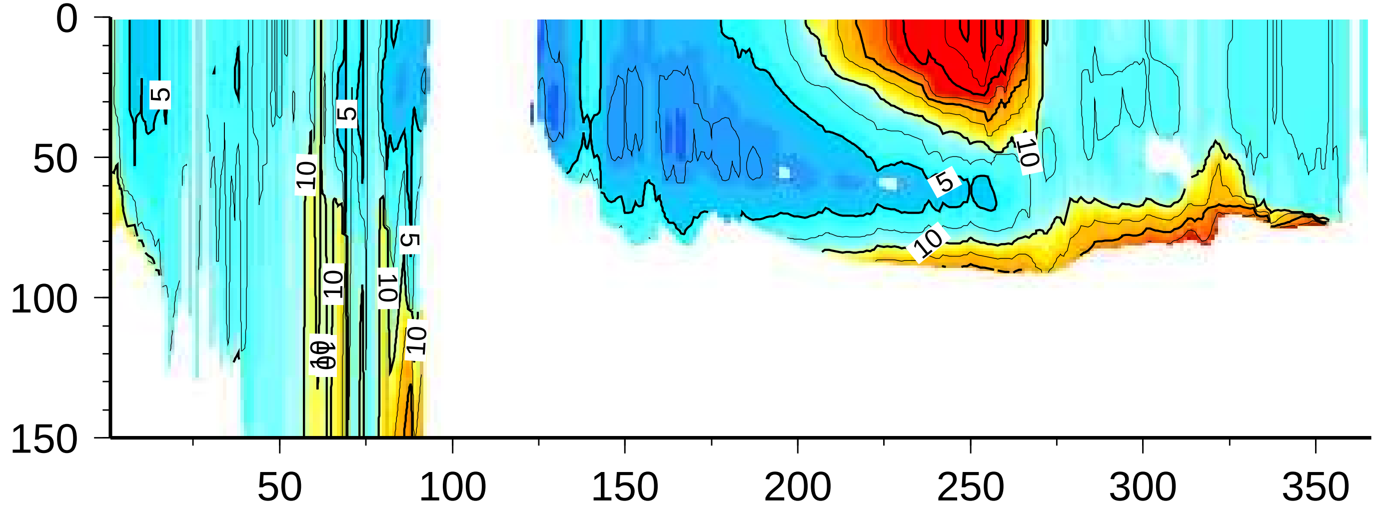
$\Rightarrow$ Age = time elapsed since the inorganic material entered the food web.

Nanophytoplankton

Concentration :



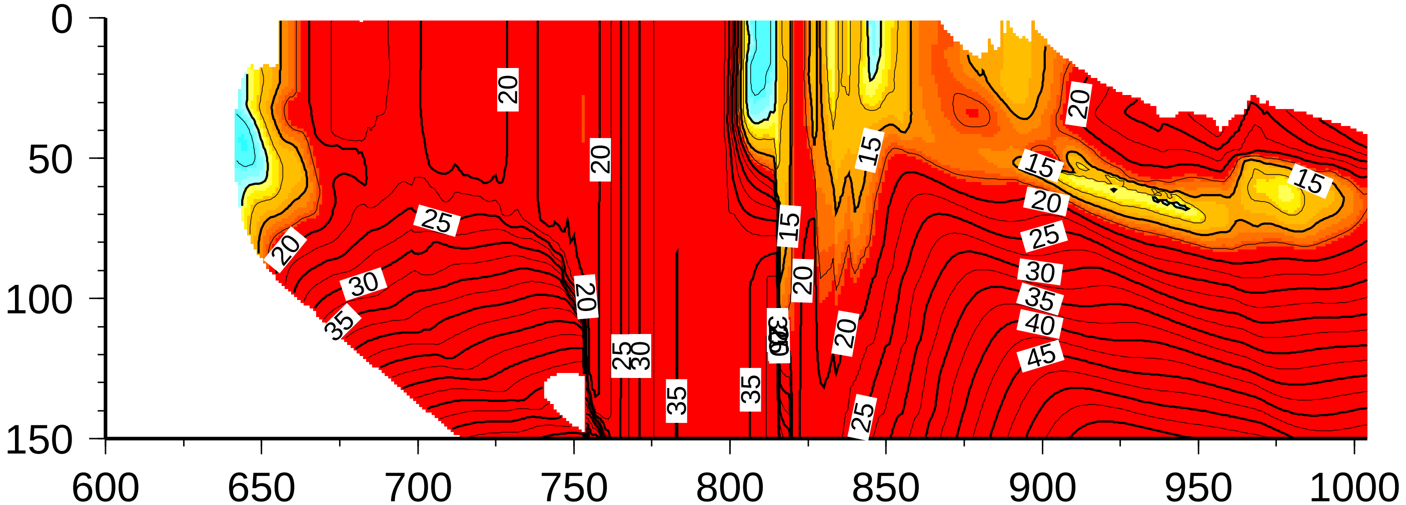
Age :



CART - 2007

Detritus

Age of POM I (days) :



$\Rightarrow$ Rate of functioning of the ecosystem and export to deep layers.

Conclusion of part I

- Non linear theory of the age of any constituent of sea water, passive or not.
- Easy to implement in numerical models.
- Can be tailored to study particular issues/problems.